

Centre Number						Candidate Number				
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Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
June 2012

Mathematics

MM04

Unit Mechanics 4

Monday 25 June 2012 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
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2	
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5	
6	
TOTAL	



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Answer **all** questions.

Answer each question in the space provided for that question.

- 1** A light rod has its ends at the points $P(-2, -1, 4)$ and $Q(4, 1, 6)$. A force $\mathbf{F}$ acts at the point M , the mid-point of PQ , where $\mathbf{F} = a\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.
- (a) Show that $\overrightarrow{PM} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$. (2 marks)
- (b) Find the moment of $\mathbf{F}$ about the point P , giving your answer in terms of a . (3 marks)
- (c) Given that the magnitude of the moment is $5\sqrt{2}$, find the two possible values of a . (4 marks)

QUESTION
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Answer space for question 1

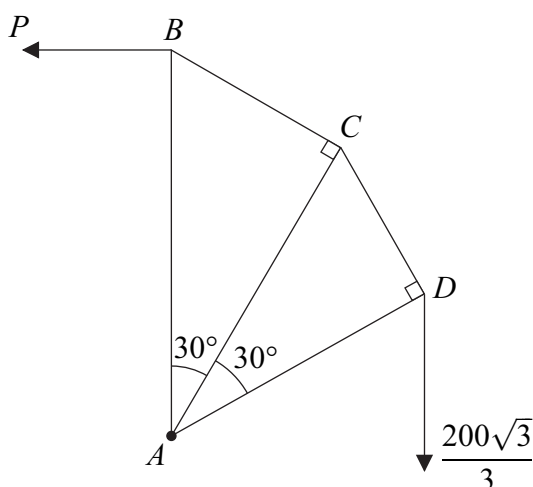


Answer space for question 1

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- 2** A framework is composed of five light smoothly-jointed rods, AB , AC , AD , BC and CD . The framework contains two right-angled triangles, ABC and ACD . The angle $BAC = \text{angle } CAD = 30^\circ$. The lengths of AB , AC and AD are $2l$, $\sqrt{3}l$ and $\frac{3l}{2}$ respectively.

The framework is in equilibrium in a vertical plane and is freely hinged at A to a fixed support. A vertical force of $\frac{200\sqrt{3}}{3}$ N acts at D . The rod AB is kept vertical by a horizontal force of magnitude P newtons, acting in the same plane as the framework at B , as shown in the diagram.



- (a) Show that $P = 75$. (3 marks)
- (b) Find the magnitudes of the forces in the rods BC and AB , stating whether each rod is in tension or compression. (6 marks)
- (c) By considering the forces perpendicular to AC at the point C , find the magnitude of the force in the rod CD . (2 marks)
- (d) For each of the rods CD , AC and AD , state whether the rod is in tension or compression. (2 marks)

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- 3** Four forces, acting in the x - y plane, act at the points with coordinates as listed below, where p and q are constants and $\mathbf{i}$ and $\mathbf{j}$ are parallel to the x -axis and y -axis respectively.

Force		Coordinates
$-2\mathbf{i}$	acting at	$(3, 3)$
$-5\mathbf{j}$	acting at	$(4, 0)$
$p\mathbf{i} + \mathbf{j}$	acting at	$(-8, -2)$
$-3\mathbf{i} - 4\mathbf{j}$	acting at	$(0, q)$

- (a) Write down the resultant of these forces in terms of p . (1 mark)
- (b) (i) Given that the system of forces is equivalent to a single force, $\mathbf{F}$, acting parallel to the y -axis, find the value of p . (2 marks)
- (ii) Given further that the equation of the line of action of $\mathbf{F}$ is $x = 3$, find the value of q . (6 marks)
- (c) The system of forces can be replaced by a single force acting at the origin together with a couple C .

Write down the magnitude of C and indicate its sense on a diagram. (2 marks)

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Answer space for question 3

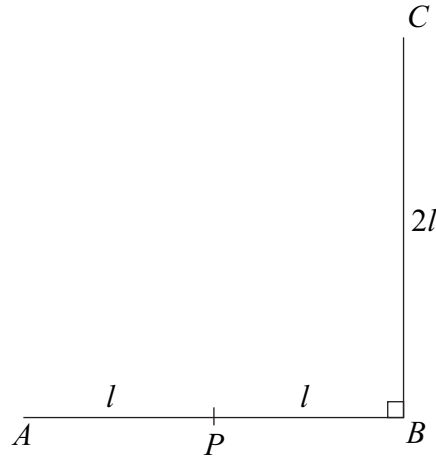
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4

Two identical uniform rods, AB and BC , are rigidly joined together to form a right angle at B . The mid-point of the rod AB is P . Each rod has mass m and length $2l$. The rods can rotate freely in a vertical plane about a horizontal axis through P , perpendicular to the plane containing ABC .



- (a) Show that the moment of inertia of the rod BC about the axis through P is $\frac{7ml^2}{3}$. (4 marks)
- (b) Particles of masses $3m$, $3m$ and $4m$ are fixed at A , B and C respectively. Find the moment of inertia of the whole system about the axis through P . (5 marks)
- (c) The system is released from rest with AB horizontal and C vertically above B . Find, in terms of g and l , the angular velocity of the system when BC is horizontal and A is vertically above B . (7 marks)

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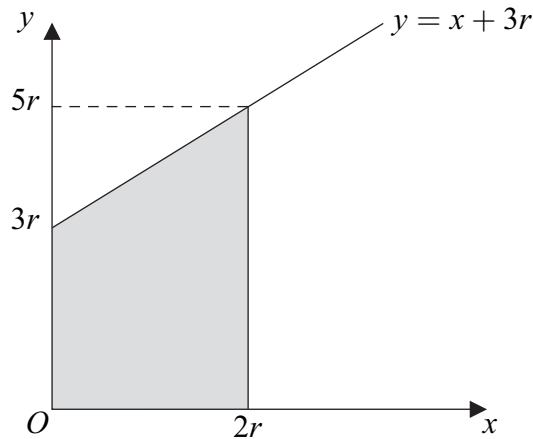
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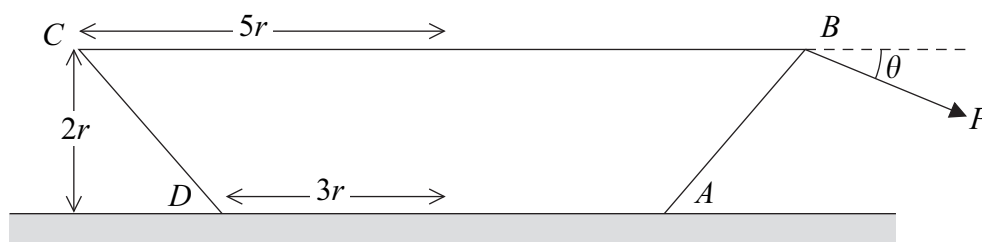


- 5 The region bounded by the line $y = x + 3r$, the y -axis, the x -axis and the line $x = 2r$ is shown in the diagram.



This region is rotated about the x -axis to form a frustum, of volume $\frac{98\pi r^3}{3}$, of a uniform solid cone.

- (a) Using integration, find the distance of the centre of mass of the frustum of the cone from O . (6 marks)
- (b) The frustum of a uniform solid cone with radii $3r$ and $5r$ and height $2r$ has weight W . The frustum stands on a horizontal surface. The diagram shows a cross-section, $ABCD$, which includes the axis of symmetry, of this frustum. A force P is applied to the point B and acts in the same plane as this cross-section at an angle of θ below the horizontal.



The frustum does not slide and is about to topple about A .

- (i) Show that

$$P = \frac{3W}{2(\cos \theta + \sin \theta)} \quad (4 \text{ marks})$$

- (ii) Find, in terms of W , the minimum possible value of P . (3 marks)

- (iii) State the value of θ for which P is a minimum. (1 mark)



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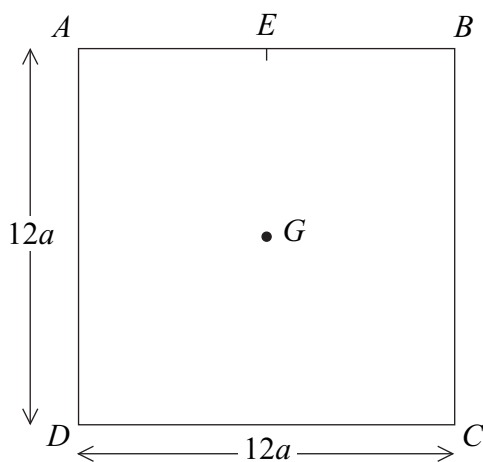
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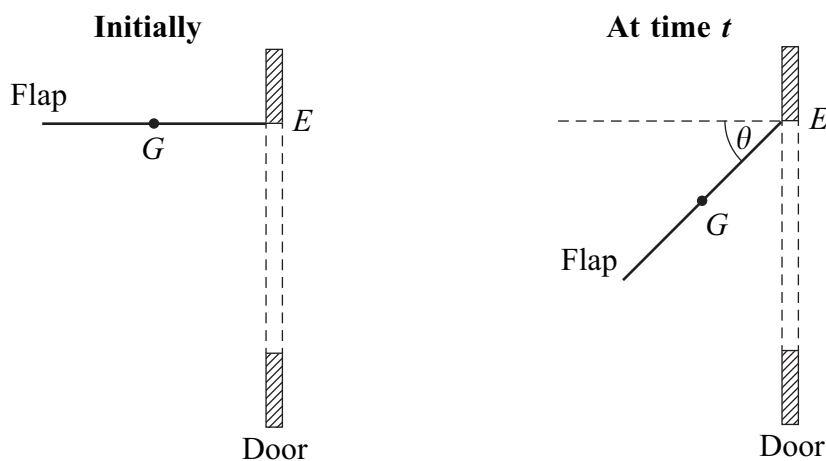
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- 6 A uniform lamina, of mass m , is in the shape of a square, $ABCD$, with each side of length $12a$. The centre of mass of the lamina is G , and the mid-point of AB is E .



- (a) Show, by using integration, that the moment of inertia of the square lamina about a fixed horizontal axis along the side AB is $48ma^2$. (5 marks)
- (b) A cat flap in a door can be modelled by the lamina described above. The flap is free to rotate about a fixed horizontal axis along AB . Initially, the flap is held at rest so that EG is horizontal, and it is then released. At time t after release, EG makes an angle θ with the horizontal.



- (i) Show that $\ddot{\theta} = \frac{g \cos \theta}{8a}$. (3 marks)
- (ii) Find, in terms of m , g and θ , the magnitude of the reaction that the flap exerts on the axis in the direction perpendicular to EG . (3 marks)
- (iii) State one improvement that could be made to the model. (1 mark)



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ANSWER IN THE SPACES PROVIDED**

